

Method of Transform

Worked Examples

William F. Barnes

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Monic and easy integer cases

Example 1.

Factor: $x^2 - 6x - 91$

$$\begin{aligned}w^2 + 6w - 91 &= (w + 3)^2 - 100 = 0 \\&\rightarrow w_{1,2} = -3 \pm 10 = \{7, -13\} \\x^2 - 6x - 91 &= x^2 + 7x - 13x - 7 \cdot 13 \\&= x(x + 7) - 13(x + 7) \\&= (x + 7)(x - 13)\end{aligned}$$

Example 2.

Factor: $x^2 + 9x + 20$

$$\begin{aligned}w^2 - 9w + 20 &= (w - 4)(w - 5) = 0 \\&\rightarrow w_{1,2} = \{4, 5\} \\x^2 + 9x + 20 &= x^2 + 4x + 5x + 4 \cdot 5 \\&= x(x + 4) + 5(x + 4) \\&= (x + 4)(x + 5)\end{aligned}$$

Example 3.Factor: $x^2 + 10x + 24$

$$w^2 - 10w + 24 = (w - 6)(w - 4) = 0$$

$$\rightarrow w_{1,2} = \{6, 4\}$$

$$\begin{aligned} x^2 + 10x + 24 &= x^2 + 6x + 4x + 6 \cdot 4 \\ &= x(x + 6) + 4(x + 6) \\ &= (x + 6)(x + 4) \end{aligned}$$

Example 4.Factor: $x^2 - 13x + 40$

$$w^2 + 13w + 40 = (w + 8)(w + 5) = 0$$

$$\rightarrow w_{1,2} = \{-8, -5\}$$

$$\begin{aligned} x^2 - 13x + 40 &= x^2 - 8x - 5x + 8 \cdot 5 \\ &= x(x - 8) - 5(x - 8) \\ &= (x - 8)(x - 5) \end{aligned}$$

Example 5.Factor: $x^2 + 32x + 192$

$$w^2 - 32w + 192 = (w - 16)^2 - 64 = 0$$

$$\rightarrow w_{1,2} = 16 \pm 8 = \{8, 24\}$$

$$\begin{aligned} x^2 + 32x + 192 &= x^2 + 8x + 24x + 8 \cdot 24 \\ &= x(x + 8) + 24(x + 8) \\ &= (x + 8)(x + 24) \end{aligned}$$

Example 6.Factor: $x^2 + 4xy - 21y^2$

$$w^2 - 4yw - 21y^2 = (w - 2y)^2 - (5y)^2 = 0$$

$$\rightarrow w_{1,2} = 2y \pm 5y = \{7y, -3y\}$$

$$\begin{aligned} x^2 + 4xy - 21y^2 &= x^2 + 7yx - 3yx - 21y^2 \\ &= x(x + 7y) - 3y(x + 7y) \\ &= (x + 7y)(x - 3y) \end{aligned}$$

Example 7.

Factor: $2x^2 - x - 6$

$$w^2 + w - 12 = (w + 4)(w - 3) = 0$$

$$\rightarrow w_{1,2} = \{-4, 3\}$$

$$\begin{aligned} 2x^2 - x - 6 &= 2x^2 - 4x + 3x - 6 \\ &= 2x(x - 2) + 3(x - 2) \\ &= (x - 2)(2x + 3) \end{aligned}$$

Example 8.

Factor: $2x^2 - 4x - 16$

$$w^2 + 4w - 32 = (w + 8)(w - 4) = 0$$

$$\rightarrow w_{1,2} = \{-8, 4\}$$

$$\begin{aligned} 2x^2 - 4x - 16 &= 2x^2 - 8x + 4x - 16 \\ &= 2x(x - 4) + 4(x - 4) \\ &= (x - 4)(2x + 4) \\ &= 2(x - 4)(x + 2) \end{aligned}$$

Example 9.

Factor: $2x^2 + 5x + 3$

$$w^2 - 5w + 6 = (w - 2)(w - 3) = 0$$

$$\rightarrow w_{1,2} = \{2, 3\}$$

$$\begin{aligned} 2x^2 + 5x + 3 &= 2x^2 + 2x + 3x + 3 \\ &= 2x(x + 1) + 3(x + 1) \\ &= (x + 1)(2x + 3) \end{aligned}$$

Example 10.

Factor: $2x^2 + 7x + 3$

$$w^2 - 7w + 6 = (w - 6)(w - 1) = 0$$

$$\rightarrow w_{1,2} = \{6, 1\}$$

$$\begin{aligned} 2x^2 + 7x + 3 &= 2x^2 + 6x + x + 3 \\ &= 2x(x + 3) + (x + 3) \\ &= (x + 3)(2x + 1) \end{aligned}$$

Example 11.Factor: $2x^2 - 7x - 4$

$$w^2 + 7w - 8 = (w + 8)(w - 1) = 0$$

$$\rightarrow w_{1,2} = \{-8, 1\}$$

$$\begin{aligned} 2x^2 - 7x - 4 &= 2x^2 - 8x + x - 4 \\ &= 2x(x - 4) + (x - 4) \\ &= (x - 4)(2x + 1) \end{aligned}$$

Example 12.Factor: $2x^2 + 9x + 9$

$$w^2 - 9w + 18 = (w - 6)(w - 3) = 0$$

$$\rightarrow w_{1,2} = \{6, 3\}$$

$$\begin{aligned} 2x^2 + 9x + 9 &= 2x^2 + 6x + 3x + 9 \\ &= 2x(x + 3) + 3(x + 3) \\ &= (x + 3)(2x + 3) \end{aligned}$$

Example 13.Factor: $3x^2 - 2x - 8$

$$w^2 + 2w - 24 = (w + 6)(w - 4) = 0$$

$$\rightarrow w_{1,2} = \{-6, 4\}$$

$$\begin{aligned} 3x^2 - 2x - 8 &= 3x^2 - 6x + 4x - 8 \\ &= 3x(x - 2) + 4(x - 2) \\ &= (x - 2)(3x + 4) \end{aligned}$$

Example 14.Factor: $3x^2 - 6x - 45 = 3(x^2 - 2x - 15)$

$$w^2 + 2w - 15 = (w + 5)(w - 3) = 0$$

$$\rightarrow w_{1,2} = -1 \pm 4 = \{-5, 3\}$$

$$\begin{aligned} 3(x^2 - 2x - 15) &= 3(x^2 + 3x - 5x - 15) \\ &= 3(x(x + 3) - 5(x + 3)) \\ &= 3(x + 3)(x - 5) \end{aligned}$$

Example 15.Factor: $3x^2 - 11x - 4$

$$w^2 + 11w - 12 = (w + 12)(w - 1) = 0$$

$$\rightarrow w_{1,2} = \{-12, 1\}$$

$$\begin{aligned} 3x^2 - 11x - 4 &= 3x^2 - 12x + x - 4 \\ &= 3x(x - 4) + (x - 4) \\ &= (x - 4)(3x + 1) \end{aligned}$$

Example 16.Factor: $4x^2 + 12x + 9$

$$w^2 - 12w + 36 = (w - 6)^2 = 0$$

$$\rightarrow w_{1,2} = 6$$

$$\begin{aligned} 4x^2 + 12x + 9 &= 4x^2 + 6x + 6x + 9 \\ &= 2x(2x + 3) + 3(2x + 3) \\ &= (2x + 3)^2 \end{aligned}$$

Example 17.Factor: $4x^2 + 13x + 10$

$$w^2 - 13w + 40 = (w - 8)(w - 5) = 0$$

$$\rightarrow w_{1,2} = \{8, 5\}$$

$$\begin{aligned} 4x^2 + 13x + 10 &= 4x^2 + 8x + 5x + 10 \\ &= 4x(x + 2) + 5(x + 2) \\ &= (x + 2)(4x + 5) \end{aligned}$$

Example 18.Factor: $4x^2 + 16x + 7$

$$w^2 - 16w + 28 = (w - 2)(w - 14) = 0$$

$$\rightarrow w_{1,2} = \{2, 14\}$$

$$\begin{aligned} 4x^2 + 16x + 7 &= 4x^2 + 2x + 14x + 7 \\ &= 2x(2x + 1) + 7(2x + 1) \\ &= (2x + 1)(2x + 7) \end{aligned}$$

Example 19.Factor: $4x^2 + 16x + 15$

$$w^2 - 16w + 60 = (w - 8)^2 - 64 + 60 = 0$$

$$\rightarrow w_{1,2} = 8 \pm 2 = \{6, 10\}$$

$$\begin{aligned} 4x^2 + 16x + 15 &= 4x^2 + 6x + 10x + 15 \\ &= 2x(2x + 3) + 5(2x + 3) \\ &= (2x + 3)(2x + 5) \end{aligned}$$

Example 20.Factor: $6x^2 - 5x - 4$

$$w^2 + 5w - 24 = (w - 3)(w + 8) = 0$$

$$\rightarrow w_{1,2} = \{3, -8\}$$

$$\begin{aligned} 6x^2 - 5x - 4 &= 6x^2 + 3x - 8x + 4 \\ &= 3x(2x - 1) + 4(2x - 1) \\ &= (2x - 1)(3x + 4) \end{aligned}$$

Example 21.Factor: $6x^2 + 11x + 4$

$$w^2 - 11w + 24 = (w - 3)(w - 8) = 0$$

$$\rightarrow w_{1,2} = \{3, 8\}$$

$$\begin{aligned} 6x^2 + 11x + 4 &= 6x^2 + 3x + 8x + 4 \\ &= 3x(2x + 1) + 4(2x + 1) \\ &= (2x + 1)(3x + 4) \end{aligned}$$

Example 22.Factor: $-6x^2 - x + 2$

$$w^2 + w - 12 = (w + 4)(w - 3) = 0$$

$$\rightarrow w_{1,2} = \{-4, 3\}$$

$$\begin{aligned} -6x^2 - x + 2 &= -6x^2 - 4x + 3x + 2 \\ &= -2x(3x + 2) + (3x + 2) \\ &= (3x + 2)(-2x + 1) \\ &= -(3x + 2)(2x - 1) \end{aligned}$$

Large AC-search

Example 23.

Factor: $10x^2 + 3x - 27$

$$w^2 - 3w + 270 = 9 \left(\left(\frac{w}{3} \right)^2 - \frac{w}{3} + 30 \right) = 0$$

$$\rightarrow w = -15$$

$$\rightarrow \alpha = \gcd(A, w), \beta = \alpha C/w = 9$$

$$\rightarrow \alpha' = 2, \beta' = -3$$

$$\begin{aligned} 10x^2 + 3x - 27 &= (\alpha x + \beta) (\alpha' x + \beta') \\ &= (5x + 9) (2x - 3) \end{aligned}$$

Example 24.

Factor: $12x^2 - 29x - 60$

$$w^2 + 29w - 720 = \left(w + \frac{29}{2} \right)^2 - \frac{841}{4} - 720 = 0$$

$$\rightarrow w_{1,2} = -\frac{29}{2} \pm \frac{61}{2} = \{16, -45\}$$

$$\begin{aligned} 12x^2 - 29x - 60 &= 12x^2 + 16x - 45x - 60 \\ &= 4x(3x + 4) - 15(3x + 4) \\ &= (3x + 4)(4x - 15) \end{aligned}$$

Example 25.

Factor: $12x^2 - 29x - 60 = (\alpha x + \beta) (\alpha' x + \beta')$

$$\alpha\alpha'\beta\beta' = w_1w_2 = -(3 \cdot 2^2) \cdot (5 \cdot 3 \cdot 2^2)$$

$$= (16) \cdot (-3^2 \cdot 5)$$

$$w_{1,2} = \{16, -45\}$$

$$\begin{aligned} 12x^2 - 29x - 60 &= 12x^2 + 16x - 45x - 60 \\ &= 4x(3x + 4) - 15(3x + 4) \\ &= (3x + 4)(4x - 15) \end{aligned}$$

Example 26.Factor: $12x^2 + 243x - 189$

$$\begin{aligned}
w^2 - 243w - 12 \cdot 189 &= \left(w - \frac{243}{2}\right)^2 - \frac{243^2}{4} - 12 \cdot 189 = 0 \\
\rightarrow w_{1,2} &= \frac{243}{2} \pm \frac{260}{2} = \{252, -9\} \\
12x^2 + 243x - 189 &= 12x^2 + 252x - 9x - 189 \\
&= 12x(x + 21) - 9(x + 21) \\
&= (x + 21)(12x - 9)
\end{aligned}$$

Example 27.Factor: $15x^2 - 16x - 15$

$$\begin{aligned}
w^2 + 16w - 15^2 &= (w + 8)^2 - 64 - 15^2 = 0 \\
\rightarrow w_{1,2} &= -8 \pm 17 = \{-25, 9\} \\
15x^2 - 16x - 15 &= 15x^2 - 25x + 9x - 15 \\
&= 5x(3x - 5) + 3(3x - 5) \\
&= (3x - 5)(5x + 3)
\end{aligned}$$

Example 28.Factor: $72x^2 + 11x - 35 = (\alpha x + \beta)(\alpha'x + \beta')$

$$\begin{aligned}
\alpha\alpha'\beta\beta' &= w_1w_2 = -(9 \cdot 8) \cdot (7 \cdot 5) \\
&= (-9 \cdot 5) \cdot (8 \cdot 7) \\
w_{1,2} &= \{-45, 56\} \\
72x^2 + 11x - 35 &= 72x^2 - 45x + 56x - 35 \\
&= 9x(8x - 5) + 7(8x - 5) \\
&= (8x - 5)(9x + 7)
\end{aligned}$$

Irrational w-solutions

Example 29.

Factor: $x^2 - 4x - 2$

$$w^2 + 4w - 2 = (w + 2)^2 - 6 = 0$$

$$\rightarrow w_{1,2} = -2 \pm \sqrt{6}$$

$$\begin{aligned} x^2 - 4x - 2 &= x^2 + w_1x + w_2x + w_1 \cdot w_2 \\ &= x(x + w_1) + w_2(x + w_1) \\ &= (x + w_1)(x + w_2) \\ &= (x - 2 + \sqrt{6})(x - 2 - \sqrt{6}) \end{aligned}$$

Example 30.

Factor: $x^2 + 6x + 2$

$$w^2 - 6w + 2 = (w - 3)^2 - 7 = 0$$

$$\rightarrow w_{1,2} = 3 \pm \sqrt{7}$$

$$\begin{aligned} x^2 + 6x + 2 &= x^2 + w_1x + w_2x + w_1 \cdot w_2 \\ &= x(x + w_1) + w_2(x + w_1) \\ &= (x + w_1)(x + w_2) \\ &= (x + 3 + \sqrt{7})(x + 3 - \sqrt{7}) \end{aligned}$$

Example 31.

Factor: $x^2 + 8x + 4$

$$w^2 - 8w + 4 = (w - 4)^2 - 12 = 0$$

$$\rightarrow w_{1,2} = 4 \pm 2\sqrt{3}$$

$$\begin{aligned} x^2 + 8x + 4 &= x^2 + w_1x + w_2x + w_1w_2 \\ &= x(x + w_1) + w_2(x + w_1) \\ &= (x + w_1)(x + w_2) \\ &= (x + 4 + 2\sqrt{3})(x + 4 - 2\sqrt{3}) \end{aligned}$$

Imaginary w-solutions

Example 32.

Factor: $3x^2 + 4x + 10$

$$w^2 - 4w + 30 = (w - 2)^2 + 26 = 0$$

$$\rightarrow w_{1,2} = 2 \pm i\sqrt{26}$$

$$\begin{aligned} 3x^2 + 4x + 10 &= 3x^2 + w_1x + w_2x + \frac{w_1w_2}{3} \\ &= x(3x + w_1) + \frac{w_2}{3}(3x + w_1) \\ &= (3x + w_1)\left(x + \frac{w_2}{3}\right) \\ &= \left(3x + 2 + i\sqrt{26}\right)\left(x + \frac{2}{3} - \frac{i\sqrt{26}}{3}\right) \end{aligned}$$